

The Pion Form Factor. Where Does It Come From ?

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Abstract

We discuss the nonleading, “soft” contribution to the pion form factor at intermediate momentum transfer within operator product expansion approach. We argue, that the corresponding contribution can temporarily **simulate** the leading twist behavior in the extent region of Q^2 : $3\text{GeV}^2 \leq Q^2 \leq 35\text{GeV}^2$, where $Q^2 F(Q^2) \sim \text{const.}$ Such a mechanism, if it is correct, would be an explanation of the phenomenological success of the dimensional counting rules (which theoretically correspond to the keeping of the asymptotically leading terms only) at available, very modest Q^2 . The relation with quark model calculation is also discussed.

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1. Introduction

The investigation of the exclusive asymptotic processes has a long history. In early seventies the famous dimensional counting rules were proposed [1]. The predictions of these rules agree well with experimental data, such as the pion and nucleon form factors, large angle elastic scattering cross sections and so on. This agreement served as a stimulus for further theoretical investigations. The modern approach to exclusive processes was started in the late seventies and early eighties [2]. We refer to the review papers [3],[4] for the detail analysis and discussions.

However, since 1981, the applicability of the approach [2]- [4] at experimentally accessible momentum transfers was questioned [5],[6]. In these papers it was demonstrated, that the perturbative, asymptotically leading contribution, is much smaller than the nonleading contributions. Similar conclusion, supporting this result, came from the different side, from the QCD sum rules, [7],[8], where the direct calculation of the form factor has been presented at $Q^2 \leq 3GeV^2$. This method, unfortunately, by some technical reasons, can not directly be applied for the analysis of the form factor $F_\pi(Q^2)$ (see, however, [9]) at larger $Q^2 \geq 3GeV^2$. Thus, the question “ what kind of contributions are responsible for the $F_\pi(Q^2)$ at $Q^2 \geq 3GeV^2$ ” can not be answered within this method.

At the same time, the information which can be extracted from the different QCD sum rules, unambiguously shows the asymmetric form of the leading twist π -meson wave function wf , [10]. The application of this wf to different amplitudes gives very sensible result at $Q^2 \simeq 10GeV^2$ and one could think that the region $3GeV^2 \leq Q^2 \leq 10GeV^2$ is the transition zone, where the asymptotically leading contribution comes into the game. However, the recent papers [11],[12] do not support this conjecture. Namely, it was shown that the inclusion of the intrinsic \vec{k}_\perp^2 - dependence and Sudakov suppression leads to the self-consistent calculation, but the obtained magnitude is too small (at least there is factor 3 in the description of the π -meson form factor at $Q^2 \simeq 10GeV^2$) with respect to the data even if ϕ_{CZ} asymmetric function is used.

Now we are ready to formulate the question, which we want to discuss in the present letter.

- If the asymptotically leading contribution can not provide the experimentally observable absolute values, than *how can one explain the very good agreement the experimental data with dimensional counting rules* ², which supposed to be valid only in the region where the leading terms dominate?

² Let us remind, that these rules unambiguously predict the dependence of amplitudes on dimensional parameters. In particular, $Q^2 F_\pi(Q^2) \simeq constant.$, $Q^4 F_p(Q^2) \simeq constant.$, $s^7 \frac{d\sigma}{dt}(\gamma p \rightarrow \pi^+ n) = f(t/s)$.. The experimental data are in a good agreement with these predictions in the large region of s, Q^2 . at very modest energy and momentum transfer.

It is clear, that the possible explanation can not be related to the specific amplitude, but instead, it should be connected, somehow, with the nonperturbative wave functions of the light hadrons ($\pi, \rho, p\dots$) which enter to the formulae for exclusive processes. The analysis of the π meson form factor, presented below supports this idea.

To anticipate the events we would like to formulate here the result of this letter. The very unusual properties of the transverse momentum distribution of the nonperturbative π meson wave function lead to the **temporarily simulation** of the dimensional counting rules by soft mechanism for the $F_\pi(Q^2)$ at the extent range of intermediate momentum transfer: $3GeV^2 \leq Q^2 \leq 35GeV^2$. In this region the soft contribution to $F_\pi(Q^2)Q^2$ does not fall-off, as naively one could expect, and we estimate it as $F_\pi(Q^2)Q^2 \simeq 0.4GeV^2$. The leading twist contribution, after Sudakov suppression, gives, according to [12], a little bit less (about $0.2GeV^2$).

2. Constraints on the nonperturbative wave function $\psi(\vec{k}_\perp^2, x)$.

First of all let me remind some essential definitions and results from [13] about nonperturbative wf . We define the pion axial wave function in the following gauge-invariant way:

$$\begin{aligned} if_\pi q_\mu \phi_A(zq, z^2) &= \langle 0 | \bar{d}(z) \gamma_\mu \gamma_5 e^{ig \int_{-z}^z A_\mu dz_\mu} u(-z) | \pi(q) \rangle \\ &= \sum_n \frac{i^n}{n!} \langle 0 | \bar{d}(0) \gamma_\mu \gamma_5 (i z_\nu \overleftrightarrow{D}_\nu)^n u(0) | \pi(q) \rangle, \end{aligned} \quad (1)$$

where $\overleftrightarrow{D}_\nu \equiv \overrightarrow{D}_\nu - \overleftarrow{D}_\nu$ and $i\overrightarrow{D}_\mu = i\overrightarrow{\partial}_\mu + gA_\mu^a \frac{\lambda^a}{2}$ is the covariant derivative. From its definition is clear that the set of different π meson matrix elements defines the nonperturbative wave function. The most important (at asymptotically high Q^2) is the part related to longitudinal distribution. The corresponding problem was studied in [10, 3]. At this paper we are mainly interested in transverse distribution part, which is very important at available, not very high Q^2 . We define the mean values of the quark transverse distribution in the following way:

$$\langle 0 | \bar{d} \gamma_\nu \gamma_5 (i \overrightarrow{D}_\mu t_\mu)^{2n} u | \pi(q) \rangle = if_\pi q_\nu (-t^2)^n \frac{(2n-1)!!}{(2n)!!} \langle \vec{k}_\perp^{2n} \rangle_A, \quad (2)$$

where transverse vector $t_\mu = (0, \vec{t}, 0)$ is perpendicular to the hadron momentum $q_\mu = (q_0, 0_\perp, q_z)$. The factor $\frac{(2n-1)!!}{(2n)!!}$ is related to the integration over ϕ angle in the transverse plane: $\int d\phi (\cos \phi)^{2n} / \int d\phi = (2n-1)!! / (2n)!!$.

We interpret the $\langle \vec{k}_\perp^2 \rangle$ in this equation as a mean value of the quark perpendicular momentum. Of course it is different from the naive, gauge dependent definition like $\langle 0 | \bar{d} \gamma_\nu \gamma_5 \partial_\perp^2 u | \pi(q) \rangle$, because the physical transverse gluon is participant of this definition. However, in QCD the only questions which allowed to be asked look like that: "what the magnitude of this gauge

invariant matrix element?”. Our definition (1), (2) does satisfy to this requirement. We believe that such definition is the useful generalization of the transverse momentum conception for the interactive quark system.

We can not prove that the mean value of the transverse momentum distribution $\langle \vec{k}_\perp^2 \rangle$, defined above in terms of QCD matrix elements coincides with the perpendicular momentum in simple quark model, which is frequently used in the phenomenological analysis³. However we would like to make such assumption in the following section devoted to the phenomenological analysis of the pion form factor. At the same time, in this section we prefer to stick around QCD with its language of operators, matrix elements and vacuum condensates in order to make difference between pure QCD results (without any additional assumptions) and phenomenological analysis (next section), where such kind of assumption, motivated by quark model, has been made.

It turns out, that the calculation of $\langle \vec{k}_\perp^{2n} \rangle$, defined in (2), can be reduced to the problem of calculating of the mixed vacuum condensates. In particular

$$\langle \vec{k}_\perp^2 \rangle = \frac{5}{36} \frac{\langle \bar{q} i g \sigma_{\mu\nu} G_{\mu\nu}^a \frac{\lambda^a}{2} q \rangle}{\langle \bar{q} q \rangle} \simeq \frac{5m_0^2}{36} \simeq 0.1 GeV^2, \quad m_0^2 \simeq 0.8 GeV^2. \quad (3)$$

$$\frac{\langle \vec{k}_\perp^4 \rangle}{\langle \vec{k}_\perp^2 \rangle^2} \simeq 3K \frac{\langle g^2 G_{\mu\nu}^a G_{\mu\nu}^a \rangle}{m_0^4} \simeq 5 \div 7,$$

where we have introduced the coefficient of nonfactorizability K ($K = 1$ if the factorization would work) of the mixed vacuum condensates of dimension seven. This coefficient can be estimated with high enough accuracy and we expect $K \simeq 2.5 \div 3$.

Few comments are in order. We **define** the nonperturbative wave function $\psi(\vec{k}_\perp^2, \xi)$, which is Fourier transform of the $\phi_A(zq, z^2)$, through its moments which can be expressed in terms of nonperturbative vacuum condensates. As is known, the condensates are defined in such a way, that all gluon's and quark's virtualities, smaller than some parameter μ (point of normalization) are hidden in the definition of the “nonperturbative vacuum matrix elements”. All virtualities larger than that should be taken into account perturbatively, see [14] for detail discussion of this problem. At the same time, there is a relation between moments and condensates, as explained above. Thus, all transverse moments are defined in the same way as condensates do. This is, actually, the **definition of the nonperturbative wave function** $\psi(\vec{k}_\perp^2, \xi)$, through its moments which can be expressed in terms of nonperturbative vacuum condensates. The important consequence of the definition

³ In order to prove such identification we need to demonstrate the confinement and chiral symmetry breaking, to calculate the meson masses and wave functions and finally, to derive an effective low-energy quark model in terms of the fundamental QCD parameters (like chiral condensate). We do not have such ambitious purposes at the moment. Instead, we assume that such kind of correspondence with intuitive quark model, takes place with high enough accuracy.

can be formulated in the following way: The nonperturbative wf even for sufficiently large \vec{k}_\perp^2 does not behave like $1/\vec{k}_\perp^2$ as is frequently assumed.

Now we want to formulate some additional constraints on the nonperturbative wave function $\psi(\vec{k}_\perp^2, x)$ in order to model it. As is known, the knowledge of a finite number of moments is not sufficient to completely determine the $\psi(\vec{k}_\perp^2, x)$; the behavior of the asymptotically distant terms is a very important thing as well. We assume, that the π -meson fills a finite duality interval in the dispersion relation at any n , where n is the order of moment: $\langle \vec{k}_\perp^{2n} \rangle$. It gives the following constraint:

- $$\int d\vec{k}_\perp^2 \vec{k}_\perp^{2n} \psi(\vec{k}_\perp^2, \xi \rightarrow \pm 1) \sim (1 - \xi^2)^{n+1}, \quad x = \frac{1 + \xi}{2} \quad .$$

For the $n = 0$ we reproduce the well-known result [10] for the ϕ wave function: $\phi(\xi \rightarrow \pm 1) = \int d\vec{k}_\perp^2 \psi(\vec{k}_\perp^2, \xi \rightarrow \pm 1) \sim (1 - \xi^2)$. This constraint is extremely important and implies that the \vec{k}_\perp^2 dependence of the $\psi(\vec{k}_\perp^2, \xi = 2x - 1)$ comes **exclusively in the combination** $\vec{k}_\perp^2/(1 - \xi^2)$ at $\xi \rightarrow \pm 1$. The byproduct of this constraint can be formulated as follows. The standard assumption on factorizability of the $\psi(\vec{k}_\perp^2, \xi) = \psi(\vec{k}_\perp^2)\phi(\xi)$ **does contradict** to the very general property of the theory formulated above.

Our last constraint comes from the analysis of the asymptotically distant terms. It was argued in [13] that the moments $\langle \vec{k}_\perp^{2n} \rangle$ at asymptotically large n behave in the following way:

- $$\int d\xi \int d\vec{k}_\perp^2 \psi\left(\frac{\vec{k}_\perp^2}{1 - \xi^2}, \xi\right) (\vec{k}_\perp^2)^n \Rightarrow 1, \quad ,$$

which means that the $\psi(\frac{\vec{k}_\perp^2}{1 - \xi^2}) \Rightarrow \delta(\frac{\vec{k}_\perp^2}{1 - \xi^2} - S)$ for sufficiently large \vec{k}_\perp^2 ⁴; the S is some input dimensional parameter, which will be expressed in terms of $\langle \vec{k}_\perp^2 \rangle$.

We want to model the simplest version of the wave function which meets all requirements formulated above. First of all, as was explained, the \vec{k}_\perp^2 dependence comes only in the combination $\psi(\frac{\vec{k}_\perp^2}{1 - \xi^2})$ and at sufficiently large \vec{k}_\perp^2 we have to have the $\delta(\frac{\vec{k}_\perp^2}{1 - \xi^2} - S)$ -like dependence. Besides that, in order to reproduce the noticeable fluctuations of the \vec{k}_\perp^2 (3), we have to spread out the distribution function between $\vec{k}_\perp^2 \sim 0$ and $\vec{k}_\perp^2 \sim S$ in such a way, that the overall area will be the same, but moments should satisfy to this requirement. In principle, it can be done in arbitrary way. The simplest way to make the wf wider is to put another δ function at $\vec{k}_\perp^2 = 0$. With these remarks in mind we propose the following "two-hump" nonperturbative wave function

⁴From the physical point of view it is clear that such δ dependence should be somehow regularized. It turns out that the large α_s^n corrections are responsible for this regularization ([15], be published). The problem of regularization is not an essential point at the moment.

which meets all requirements discussed above:

$$\psi(\vec{k}_\perp^2, \xi) = [A\delta(\frac{\vec{k}_\perp^2}{1-\xi^2} - S) + B\delta(\frac{\vec{k}_\perp^2}{1-\xi^2})][g(\xi^2 - \frac{1}{5}) + \frac{1}{5}]$$

$$\langle \vec{k}_\perp^4 \rangle \simeq 5\langle \vec{k}_\perp^2 \rangle^2 \Rightarrow A = 7/8, \quad A + B = \frac{15}{4} \quad g = 1, \quad S = \frac{15}{2}\langle \vec{k}_\perp^2 \rangle \simeq 0.8 GeV^2$$

Few comments are in order. First, this is not supposed to be a realistic model. But we expect that this function illustrates features which $\psi(\vec{k}_\perp^2, \xi)$ probably does exhibit. We put the common factor $[g(\xi^2 - \frac{1}{5}) + \frac{1}{5}]$ in the front of the formula in order to reproduce the light cone $\phi(\xi)$ function with arbitrary $\langle \xi^2 \rangle$. For $g = 0$ it corresponds to the asymptotic $wf : \phi(\xi) = 3/4(1 - \xi^2)$. For $g = 1$ we reproduce the $\phi(\xi)_{CZ} = 15/4(1 - \xi^2)\xi^2$ with $\langle \xi^2 \rangle \simeq 0.43$.

Before to consider some applications, let me stress, that we discuss in this letter some qualitative properties of the wf . Thus, the model wf proposed above should be considered as illustrative example only. We try to emphasize on the very unusual qualitative feature of the model. We expect, however, that this feature will survive in the full picture, after physical regularization will be made.

3. The Pion Form factor.

The starting point is the famous Drell-Yan formula [16] (for modern, QCD- motivated employing of this formula, see [4]), where the $F_\pi(\vec{q}_\perp^2)$ is expressed in terms of full wave functions:

$$F_\pi(\vec{q}_\perp^2) = \int \frac{dx_1 d^2 \vec{k}_\perp}{16\pi^3} \Psi_{BL}^*(x_1, \vec{k}_\perp + x_2 \vec{q}_\perp) \Psi_{BL}(x_1, \vec{k}_\perp), \quad (4)$$

where $q^2 = -\vec{q}_\perp^2$ is the momentum transfer. In this formula, the $\Psi_{BL}(x_1, \vec{k}_\perp)$ is the full wave function; the perturbative tail of $\Psi_{BL}(x_1, \vec{k}_\perp)$ behaves as α_s/\vec{k}_\perp^2 for large \vec{k}_\perp^2 and should be taken into account explicitly in the calculations. This gives the one-gluon-exchange (asymptotically leading) formula for the form factor in terms of soft pion wf with removed perturbative tail [2].

Let us remind, that the formula (4) takes into account only the valence Fock states. Besides that, \vec{k}_\perp^2 in this formula is the usual (not covariant) perpendicular momentum of the constituents, and not the mean value $\langle \vec{k}_\perp^2 \rangle$ defined in QCD, as a gauge invariant object. However we make the **assumption** that it is one and the same variable. The physics behind of it can be explained in the following way.

In the formula (4) we effectively take into account some gluons (not all of them), which inevitably are participants of our definition of wf . These gluons mainly carry the transverse momentum (which anyhow, does not exceed QCD scale of order $\mu \sim 1 GeV$) or small ammount of longitudinal momentum. The contributions of the gluons carrying the *finite* longitudinal

momentum fraction are neglected in (4). This is the main assumption. It can be justified by the direct calculation [3] of quark-antiquark-gluon (with finite momentum fraction) contribution to π meson form factor at large Q^2 within the standard technique of the operator product expansion. By technical reasons the corresponding calculation has not been completed, however it was found that the characteristic scale which enters into the game is of order 1GeV^2 . Thus, it is very unlikely to expect that these contributions might be important at $Q^2 \sim 10\text{GeV}^2$. The second calculation, which confirms this point, comes from the light cone QCD sum rules [17]. This is almost model independent calculation demonstrates that the quark-antiquark-gluon (with finite momentum fraction) contribution does not exceed 20% at available Q^2 .

Thus, we expect, that by taking into account the only "soft" gluon contribution (hidden in the definition of \vec{k}_\perp^2 (2)), we catch the main effect. Again, there is no proof for that within QCD, and the only argumentation which can be delivered now in favor of it, is based on the intuitive picture of quark model, where current quark and soft gluons form a constituent quark with nonzero mass and with original quantum numbers. No evidence where a gluon would play the role of a valent participant with a finite amount of momentum, is found.

From the viewpoint of the operator product expansion, the assumption formulated above, corresponds to **summing up** a subset of higher-dimension power corrections. This subset actually is formed from the infinite number of soft gluons and unambiguously singled out by the definition of nonperturbative wf (2).

In the following, we preserve the notation $\Psi_{BL}(x_1, \vec{k}_\perp)$ for the nonperturbative, soft part only. It should not confuse the reader.

The formula (4) is written in terms of Brodsky and Lepage notations [4]; the relation to our wave function $\psi(\xi, \vec{k}_\perp)$ looks as follows:

$$\Psi_{BL}(x_1, \vec{k}_\perp) = \frac{f_\pi 16\pi^2}{\sqrt{6}} \psi(\xi, \vec{k}_\perp), \quad \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \Psi_{BL}(x_1, \vec{k}_\perp) = \frac{f_\pi}{\sqrt{6}} \phi(\xi), \quad (5)$$

where $f_\pi = 133\text{MeV}$. There are two, physically different contributions to the form factor. We want to separate them and discuss independently.

The first contribution is the standard one, in a sense that it is determined by the \vec{k}_\perp^2 -distribution about $\vec{k}_\perp^2 = 0$ region. It is proportional to the B^2 (we call the corresponding contribution F_π^{BB}) and actually depends very strongly on regularization procedure. However, at $\vec{q}_\perp^2 \simeq \text{few GeV}^2$ this contribution begins to fall off very rapidly because of the proportionality to $c \langle \vec{k}_\perp^2 \rangle^2 / \vec{q}_\perp^4$ with some coefficient c , determined by characteristic scale of the wf . Besides that, we expect that all Fock states are equally important in this region and thus, any predictions are model dependent.

However, to give some qualitative insight on the behavior of the contribution of this kind, we estimate it in the following way. We regularize our δ functions in the simplest way- we shift its argument: $\delta(\frac{\vec{k}_\perp^2}{1-\xi^2}) \rightarrow \delta(\frac{\vec{k}_\perp^2}{1-\xi^2} - m^2)$, by

introducing the new effective phenomenological parameter m^2 . This parameter effectively describes the \vec{k}_\perp^2 -distribution about zero point and somehow related to $\langle \vec{k}_\perp^2 \rangle$. This regularization is still not sufficient for the calculating of the integral (4) at $q = 0$. But we do not need it, because we are not going to apply this method for the describing of the form factor in the vicinity of $q = 0$. Nevertheless, we expect, that at $\vec{q}_\perp^2 \gg m^2$ only the integral characteristic does matter. Have these few comments in mind, we estimate the parameter m^2 from the requirement of coincidence theoretical formula (4) with the experimental data at $\vec{q}_\perp^2 \simeq 3GeV^2$: $F_\pi \vec{q}_\perp^2 \simeq 0.4GeV^2$. It gives $m^2 \simeq (0.22GeV)^2$ for our set of parameters (4). We display the result of this contribution to the form factor $\vec{q}_\perp^2 F_\pi(\vec{q}_\perp^2)$ in Fig. 1 by small-dashed curve. We took into account in this numerical calculation the evolution of the wf which determined by parameter $g(q^2)$ [2]-[4]:

$$g(q) = g(\mu) \left(\frac{\alpha_s(q)}{\alpha_s(\mu)} \right)^{50/81}, \quad g(\mu) = 1, \quad \mu = 0.5GeV, \quad \Lambda_{QCD} = 100MeV. \quad (6)$$

Let me stress again: we are not pretending to have made a reliable calculation of the form factor here; we displayed this contribution only for the **illustrative** purposes. The main feature of this contribution– it gives very reasonable magnitude for the intermediate region about few GeV^2 and it starts to fall off very quickly at $\vec{q}_\perp^2 \geq \text{few } \langle \vec{k}_\perp^2 \rangle$. We expect that **any** reasonable, well localized wf with the scale $\sim \langle \vec{k}_\perp^2 \rangle$ leads to the same behavior. We will not discuss this standard contribution any more.

Currently, much more interesting for us, is the interference term, proportional to AB . It is clear, that with increasing of \vec{q}_\perp^2 , the contribution coming from the overlap of the δ functions starts to grow. Because the δ functions are well separated from each other on the value of order $S \simeq 0.8GeV^2$, we expect that this contribution will start to increase at high enough $\vec{q}_\perp^2 \gg 4S \simeq 4GeV^2$. This contribution almost model-independent in a sense that it does not depend on the infrared regularization parameter m^2 which we introduced before, provided that $m^2 \ll S$. Let me remind at this point, that our parameters S and $\langle \vec{k}_\perp^2 \rangle$ are not the new phenomenological input parameters, but have been derived (3, 4) in terms of the vacuum characteristics, which are known independently.

With these remarks in mind, we can explicitly calculate this interference contribution:

$$F_\pi^{AB}(\vec{q}_\perp^2) = AB \frac{2\pi^2 f_\pi^2}{3S} (1 - \xi_0^2)^2 [g(\vec{q}_\perp^2) \xi_0^2 + \frac{1}{5} (1 - g(\vec{q}_\perp^2))]^2, \quad \xi_0 \equiv \frac{\vec{q}_\perp^2 - 4S}{\vec{q}_\perp^2 + 4S}. \quad (7)$$

We display the corresponding contribution to the $\vec{q}_\perp^2 F_\pi(\vec{q}_\perp^2)$ in Fig. 1 by the large-dashed curve. The solid line on this picture corresponds to the total result.

Few comments are in order. As was expected, the interference term starts to **grow** at high enough momentum transfer. This is the **main qualitative**

effect. The contribution under consideration is subject to Sudakov corrections. An estimate of these corrections reveals that they are small enough. The reason for that is the large scale $4S$ which enter to the formula (7); thus the ξ_0 is far away from the end point region ± 1 even at large \vec{q}_\perp^2 . As the second remark, we note, that the form factor is the integral characteristic and so, we expect that any reasonable regularization of δ functions which respect constraints, will not change the F_π^{AB} drastically.

At the same time we want to note that the result strongly depends on $\langle \xi^2 \rangle$. We display on Fig.2 three different curves for $\vec{q}_\perp^2 F_\pi(\vec{q}_\perp^2)$ which correspond to different choice of $\langle \xi^2 \rangle$. The small-dashed curve corresponds to the asymptotic wf with $\langle \xi^2 \rangle = 0.2$ and $g(\mu) = 0$. The large-dashed curve corresponds to $\langle \xi^2 \rangle = 0.3$, $g(\mu) = 0.44$. The solid line corresponds to $\langle \xi^2 \rangle = 0.4$, $g(\mu) = 0.88$. Let us note, that in these calculations we did not change the infrared regulator, m^2 which is an effective parameter for calculation of the “standard” F_π^{BB} contribution. But we made all necessary substitutions related to changing of the parameter $g(\mu)$ in eq.(4). We observe the strong dependence on parameter $\langle \xi^2 \rangle$. In particular, at $\langle \xi^2 \rangle = 0.4$, the soft contribution to the form factor at $\vec{q}_\perp^2 = 10 GeV^2$ is equal to $\vec{q}_\perp^2 F_\pi(\vec{q}_\perp^2) \simeq 0.4$. For the asymptotic wf with $\langle \xi^2 \rangle = 0.2$ we have $\vec{q}_\perp^2 F_\pi(\vec{q}_\perp^2) \simeq 0.1$. These estimations are in a very good agreement with absolutely independent recent calculations [17], who used a quite different method.

4. Summary and Outlook.

We hope we proposed the new “soft” mechanism which could be responsible for the explanation of the phenomenological success of the dimensional counting rules. The main idea is based on the very unusual feature (noticeable fluctuations of the transverse momentum) of the wave function . It leads to the “two-hump” shape for the nonperturbative wf in the transverse direction and to the appearing of the **new** scale ($S = 0.8 GeV^2$) in the problem, in addition to the standard low energy parameter $\langle \vec{k}_\perp^2 \rangle \simeq 0.1 GeV^2$ ⁵. As the consequence of it, the increasing of the \vec{q}_\perp^2 in the pion form factor leads to the increasing of overlap of these humps. It gives the growing contribution at intermediate \vec{q}_\perp^2 (parametrically it falls off very quickly, of course). Together with the standard, decreasing contribution, it could **simulate** the leading twist behavior in the extent region of \vec{q}_\perp^2 . Our conclusions essentially support the picture described in [18], but from the quite different side.

It would be interesting to check the conjecture formulated above (about an equivalence of the transverse momentum defined in terms of the gauge

⁵ In the standard quantum mechanics it is very unlikely to have the two-particle lowest state wave function with such unusual form. However, we consider our wf as an effective one, which counts infinite number of soft gluons. The information about them is hidden into the definition of the effective transverse momentum. We believe that precisely these gluons are responsible for the making constituent quark from the current massless quarks and gluons.

invariant operators in QCD and perpendicular momentum in quark model) in the phenomenological analysis for the different amplitudes. Besides that, we would expect that the same feature might occur in the nucleon wf and we hope to discuss it somewhere else. I thank Tolya Radyushkin and Stan Brodsky for useful comments and insights on the number of subjects related to this letter.

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FIGURES

Fig.1 Small-dashed line is the F^{BB} contribution to the pion form factor $\vec{q}_\perp^2 F_\pi^{BB}(\vec{q}_\perp^2)$. Large-dashed line is the interference term, $\vec{q}_\perp^2 F_\pi^{AB}(\vec{q}_\perp^2)$. Solid line is the total result $\vec{q}_\perp^2 F_\pi(\vec{q}_\perp^2)$ for $g(\mu) = 1$.

Fig.2 Small-dashed line is the $\vec{q}_\perp^2 F_\pi(\vec{q}_\perp^2)$ at $\langle \xi^2 \rangle = 0.2$; Large-dashed line corresponds to $\langle \xi^2 \rangle = 0.3$; Solid line is the result for $\langle \xi^2 \rangle = 0.4$.

This figure "fig1-1.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9402280v3>

This figure "fig1-2.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9402280v3>

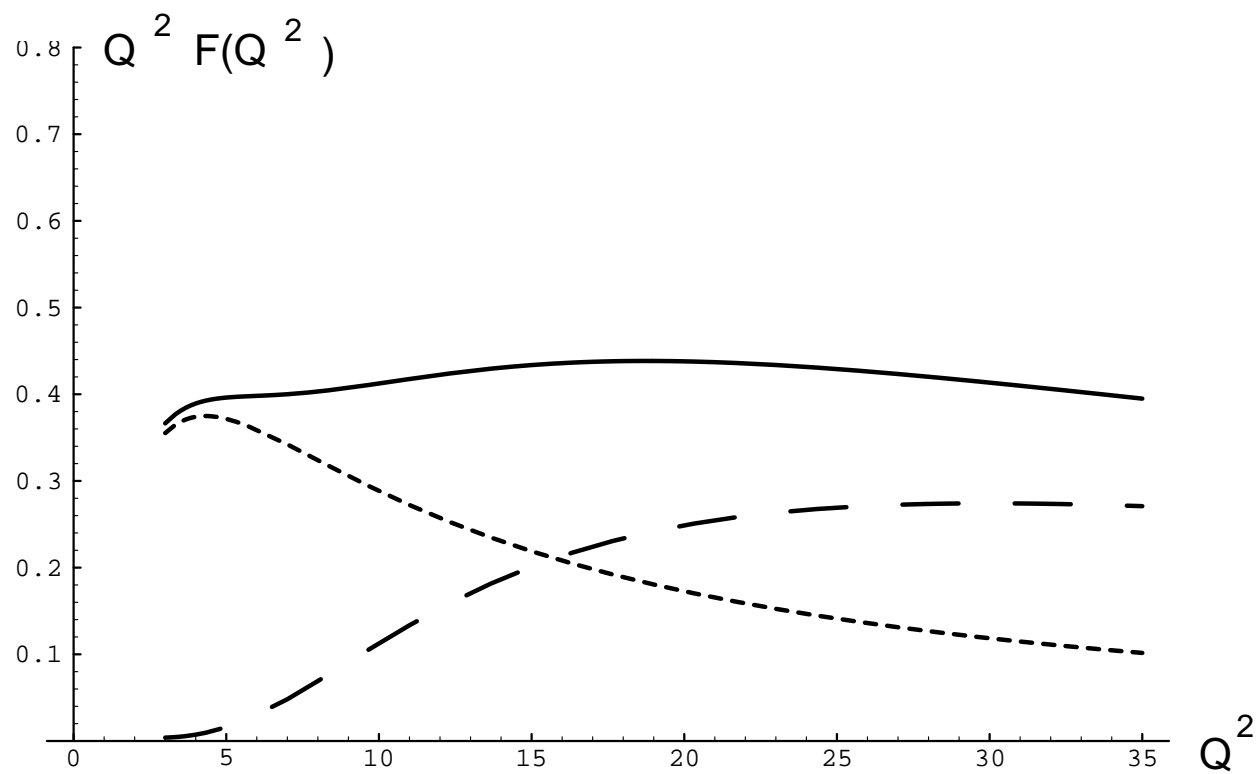


Fig.1

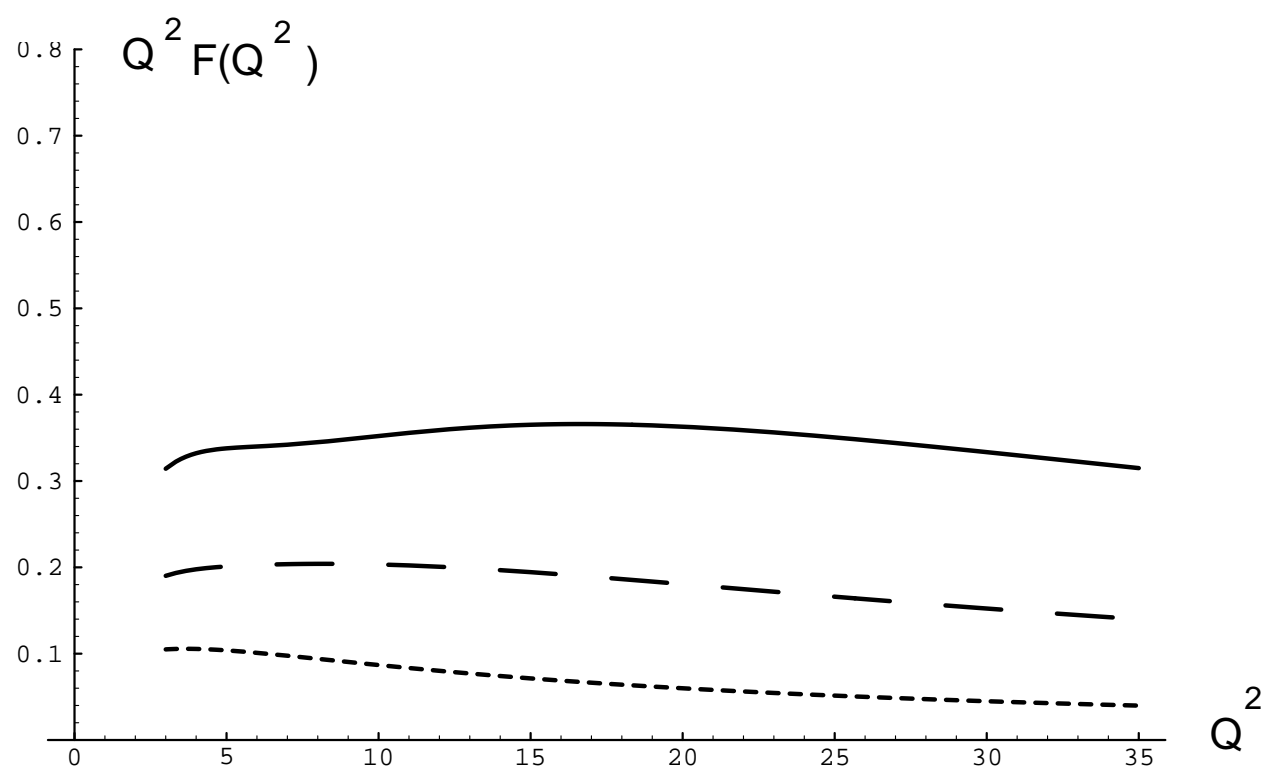


Fig.2